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# Assessing the impact of thermal feedback and recycling in open-loop groundwater heat pump (GWHP) systems: a complementary design tool

Ellen Milnes · Pierre Perrochet

**Abstract** *Thermal feedback and thermal recycling* in open-loop groundwater heat pump (GWHP) systems occurs when a fraction of the injected water in a well doublet returns to the production well. They reflect two different mathematical representations of the same physical process. Thermal feedback assumes a constant injection temperature, while thermal recycling couples the injection and production temperatures by a constant temperature difference. It is shown that thermal feedback, commonly used in GWHP design, and recycling reflect two thermal end-members. This work addresses the coupled problem of thermal recycling, which is, so far, the missing link for complete GWHP assessment. An analytical solution is presented to determine the return-flow fraction in a well doublet and is combined with a heat-balance calculation to determine the steady-state well temperatures in response to thermal feedback and recycling. This is then extended to advective-dispersive systems using transfer functions, revealing that the well temperatures in response to thermal feedback and recycling are functions of the capture probability. Conjunctive interpretation of thermal feedback and recycling yields a novel design approach with which major difficulties in the assessment of the sustainability of GWHP systems can be addressed.

**Keywords** Groundwater heat pump · Thermal feedback · Thermal recycling · Analytical solutions · Numerical modeling

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## Introduction

Open-loop groundwater heat pump (GWHP) schemes in shallow aquifers are a wide-spread renewable energy source for the heating/cooling of buildings (Banks 2009). Such systems draw groundwater from a production well and pass it through a heat exchanger before discharging it into one or several injection wells at a different temperature, leading to a thermal plume. To ensure sustainability in terms of performance, design of GWHP schemes relies upon operation within a narrow temperature range (Rafferty 1996). Since seasonal fluctuations of groundwater temperature are generally small, aquifers are ideal energy sources. If the groundwater temperature at a production well changes beyond the range for which the system was designed, the performance will decrease and may even lead to system failure. Hence, temperature changes induced by thermal plumes may pose an external risk to downstream users, or an internal risk to the sustainability of the GWHP system, due to *thermal feedback and thermal recycling* (Banks 2009). Thermal feedback and thermal recycling are two different mathematical representations of the same physical process. Today, most GWHP design approaches are based on the assessment of the impact of thermal feedback only, while thermal recycling, being a coupled process, is widely neglected.

Thermal feedback is a process that takes place when the hydraulic conditions of a well doublet are such that a thermal plume propagates back towards the production well, reducing or increasing the temperature of the captured groundwater. Such hydraulic conditions occur when the well separation between the production and injection well is not large enough, which is often the case in densely populated urban areas. Numerous analytical approaches have been developed over the past few decades describing different aspects of thermal feedback (e.g. Gringarten and Sauty 1975; Lippmann and Tsang 1980; Clyde and Madabhushi 1983; Luo and Kitanidis 2004), which are widely used by practitioners during the design of GWHP schemes to assess their sustainability. In order to simplify the mathematical description, the main assumption of these approaches is that the temperature at

the injection well remains constant, even after arrival of the thermal plume at the production well. This ignores the operational mode of heat exchangers, which is linked to a temperature difference between the production and extraction well. Thus, a thermal plume assessed with these mathematical approaches will lead to an identical thermal plume as for a well doublet without any thermal feedback and is therefore considered the ‘best-case’ scenario in terms of thermal impact on the aquifer, although it is incompatible with, and the ‘worst-case’ in terms of, the designed heat load requirement.

Thermal recycling, as opposed to thermal feedback, extends the process of the heat-plume arriving at the production well (thermal feedback) by coupling the injection and production well temperatures, thereby mimicking the process across the heat exchanger. This implies that whenever the production well temperature changes, the injection well temperature changes in parallel. Although this coupled process has been identified and discussed in literature (e.g. Warner and Algan 1984) and more recently commented upon by Banks (2009), it has not been described mathematically. In order to deal with the coupled character of this process mathematically, a constant temperature difference  $\Delta T$  across the heat exchanger between the production and injection wells is assumed in this work. This assumption is valid within the temperature range for which the system was designed. However, if the production-well temperature changes beyond this operational limit, the temperature difference  $\Delta T$  across the heat exchanger will no longer remain constant, but will be reflected in a decrease of the system performance (Rafferty 1996; Zhao et al. 2003). Hence, forcing  $\Delta T$  to remain constant will necessarily lead to a ‘worst-case’ temperature impact on the production well and aquifer, indicating system failure or performance loss if the temperature changes beyond the limits for which the system was designed. On the other hand, enforcing a constant  $\Delta T$  reflects a heat exchanger operating optimally under all conditions, and will therefore yield the highest possible extracted heat load (‘best-case’ heat load requirement).

The usefulness of these two different mathematical representations, thermal feedback and thermal recycling, is that each of them reflects a diametrically different and asymptotic situation (end-members), defining the range within which realistic situations can be expected to be situated. Since commonly used design approaches only assess thermal feedback (one end-member), the contribution of this work lies in presenting a mathematical approach to assess the second end-member, thermal recycling, thereby providing the missing link for conjunctive assessment of GWHP systems.

In line with analytical solutions for thermal feedback, a straightforward mathematical approach is presented to assess the potential impact of thermal recycling. It is first developed based on a purely advective heat mass-balance combined with an analytical solution quantifying the return-flow fraction in a well doublet. It is shown that the production-well

temperature, in response to thermal feedback and recycling, is a function of the return-flow fraction between the injection and production wells and the temperature difference across the heat exchanger. The solution is then extended to advective-dispersive systems. By making use of transfer functions, it is shown that, for thermal feedback and thermal recycling, the production-well temperatures at steady state are functions of the capture probability. Being simple in its application, the presented conjunctive assessment approach provides a novel and complementary design tool for practitioners, useful for the design of GWHP systems. It also yields new insights on thermal recycling as a coupled process.

### Thermal feedback versus thermal recycling: two end-members

The two mathematical approaches, thermal feedback and thermal recycling, addressing the process of a thermal plume propagating back to a production well in a well doublet, only differ in the way the temperature boundary condition is defined at the injection well. However, this difference has major implications for the resulting thermal impact on the production well and on the aquifer, as well as on the heat-load requirement.

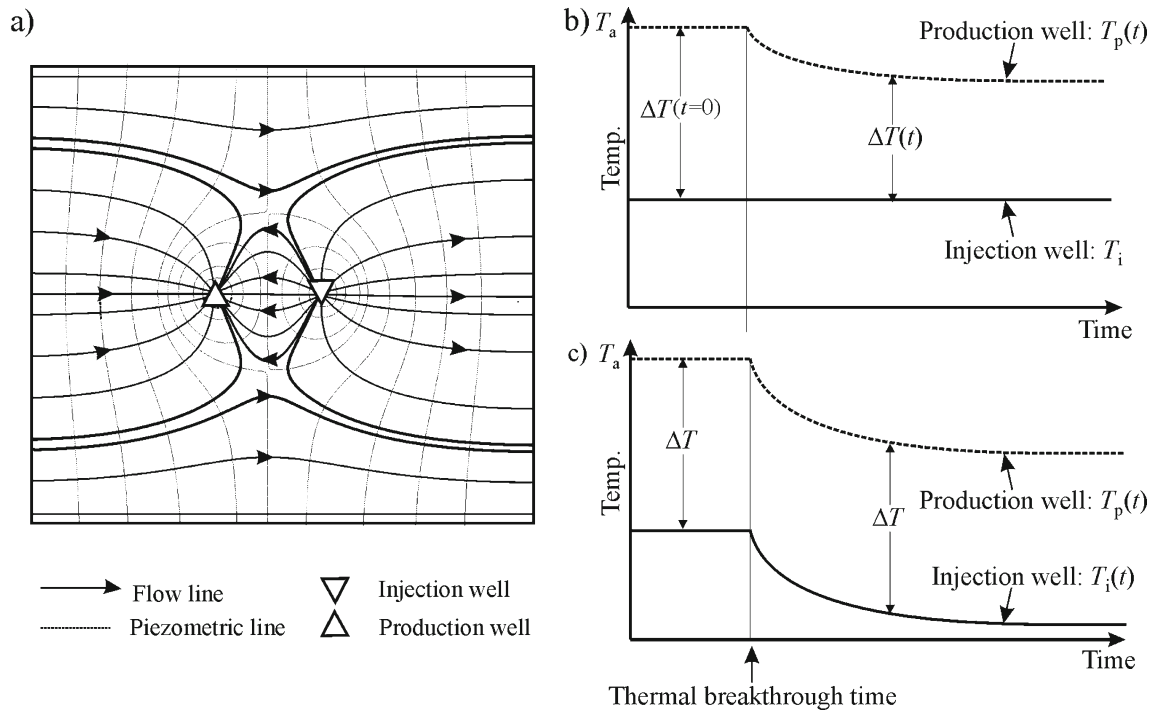
The mathematical description of thermal feedback relies on a constant injection well temperature  $T_i$ . The injection well temperature is thereby completely decoupled from the production-well temperature, which is only correct in cases when no return flow takes place. The initial ambient groundwater temperature  $T_a$  combined with the initial (design) temperature difference across the heat exchanger  $\Delta T(t=0)$  defines the constant injection temperature as follows:

$$T_i = T_a + \Delta T(t = 0) \quad (1)$$

Due to the simplicity in handling the injection well temperature boundary condition according to Eq. (1), common mathematical and numerical approaches used to model the impact of GWHP generally adopt this definition (e.g. Gringarten and Sauty 1975; Lippmann and Tsang 1980; Ferguson 2006).

Figure 1a schematically illustrates a homogeneous two-dimensional (2D) horizontal aquifer with a single well doublet with hydraulic steady-state conditions. Figure 1b shows the temperature evolution resulting from thermal feedback at the injection and production wells. The temperature at the injection well  $T_i$  remains constant, even after breakthrough of the thermal plume at the production well, which leads to a variable temperature difference  $\Delta T(t)$  between the production and injection wells across the heat exchanger.

To illustrate this, one may consider the hydraulically most extreme case, in which all injected water returns to the production well. In this case, the production-well temperature  $T_p$  would eventually equal



**Fig. 1** a Schematic illustration of a 2D horizontal, homogeneous aquifer with a regional flow from west towards east, with a single well doublet. Graphs showing the temperature evolutions of the injection and production wells for b thermal feedback and c thermal recycling. For thermal feedback (b) there is a constant injection-well temperature  $T_i$  and a variable temperature difference  $\Delta T(t)$  leading to production-well temperature  $T_p(t)$ ; and for thermal recycling (c) there is a variable injection temperature  $T_i(t)$  caused by variable production-well temperature  $T_p(t)$  and constant temperature difference  $\Delta T$

the constant injection-well temperature  $T_i = T_p$  when plume-propagation has reached steady-state conditions. As a result, this yields a heat-load requirement of zero, since  $\Delta T(t = \infty) = 0$  across the heat exchanger. Hence, the temperature difference  $\Delta T(t)$  between the injection and production wells, according to the mathematical description of thermal feedback (Eq. 1), is a function of the constant injection temperature  $T_i$ , and in no way related to the processes across the heat exchanger and to heat-load requirements. As a consequence of this, the injection-well temperature cannot drop below the initial injection temperature, although this may occur in reality. Therefore, mathematical approaches addressing thermal feedback can be considered as an upper bounding end-member, yielding the best case in terms of temperature impact on the production well and on the aquifer, while yielding a worst case with respect to the heat load requirement.

Thermal recycling, as opposed to thermal feedback, addresses the coupled character between the injection and production-well temperature by defining the injection well temperature  $T_i(t)$  as a function of the production-well temperature  $T_p(t)$ , and by simplifying the process across the heat exchanger by a constant temperature difference  $\Delta T$ , so that

$$T_i(t) = T_p(t) + \Delta T \quad (2)$$

Figure 1c shows the temperature evolution at the production and injection wells in response to thermal

recycling, according to Eq. (2). The temperature difference between the injection and production wells remains constant at all times and results in lower temperatures as compared to the temperatures in response to thermal feedback (Fig. 1b).

Considering the same hydraulic case as discussed in the preceding, where return flow within the well doublet is complete, thermal recycling will result in temperatures  $T_i(t = \infty)$  and  $T_p(t = \infty)$  tending towards  $\pm \infty$ , while the temperature difference across the heat exchanger  $\Delta T$  and the extracted heat load will remain constant, according to Eq. (2). The main justification of a constant temperature difference  $\Delta T$  across the heat exchanger is that it yields the second end-member which is directly associated with the thermal feedback end-member. Hence, in terms of thermal impact on the production-well temperature and the aquifer, thermal recycling will yield the worst-case scenario, while in terms of heat load requirements, it yields the best-case scenario.

A realistic impact of thermal feedback and recycling on the temperature evolution at the wells is closely linked to the characteristics of the heat exchanger. With a wide range of different systems, no universally applicable relationship can be established, linking the production and injection well temperatures. The real impact of return flow in a GWHP system may therefore reasonably be expected somewhere between the two end-members, i.e. thermal feedback and thermal recycling.

## Steady-state return flow in a well doublet: thermal-feedback versus thermal-recycling production temperatures

Simplified analytical approaches have been used by practitioners to assess and quantify the impact of thermal feedback on the sustainability of GWHP schemes. Although such systems are known to be highly dynamic with considerable seasonal variations, steady-state approaches are very useful as rapid assessment tools. They allow valuable comparative analysis of the asymptotic behaviour of such systems for multiple variable configurations. The fact that GWHP systems are advection dominated, has led to numerous purely advective analytical solutions, widely used by practitioners due to their simplicity (e.g. Lippmann and Tsang 1980; Clyde and Madabhushi 1983; Banks 2007, 2008).

The first step in the assessment and design of GWHP schemes is usually carried out following the classical approach considering pure advection. In a well doublet aligned in a 2D homogeneous flow-field, with Darcy strength  $q$  and thickness  $b$ , identical extraction/injection rates  $Q_p=Q_i=Q$ , and an injection well located downstream at the separation distance  $L$ , advective return flow between the injection and production wells takes place if (e.g. Banks 2009)

$$X = \frac{2Q}{\pi b q L} > 1 \quad (3)$$

This condition arises from the classical analytical solution for a well doublet (see Appendix) and allows design of a well separation in order to prevent return flow from occurring. Due to space constraints in densely inhabited urban areas, the well separation  $L$  in Eq. (3) can often not be chosen large enough with respect to the required extraction rate  $Q$  to prevent thermal feedback and thermal recycling.

Hence, when the condition  $X > 1$  is met, thermal breakthrough is likely to occur after a specific time of operation, the quantification of which is generally based on the hydraulic breakthrough-time (see Appendix) scaled by a retardation factor corresponding to the ratio between rock matrix and groundwater heat capacities (e.g. Gringarten and Sauty 1975; Lippmann and Tsang 1980). Instructive in itself, this thermal breakthrough-time is, however, not sufficient for evaluating the long-term effects of return flow on the sustainability of a GWHP system. To do so, it is necessary to quantify the fraction  $\alpha$  of the injected flow rate returning to the production well. Not found in published literature, an analytical expression for this fundamental quantity is derived (see Appendix), yielding

$$\alpha = \frac{2}{\pi} \left( \tan^{-1}(\sqrt{X-1}) - \frac{\sqrt{X-1}}{X} \right) \quad (4)$$

The return-flow fraction  $\alpha$  given in the preceding allows a simplified, purely advective heat-balance calculation for the production well at steady state, consisting of

two components associated to the flow rate: a return-flow component  $\alpha Q$  with the injection well temperature  $T_i$ , and a far-field component  $(1-\alpha)Q$  with the ambient temperature  $T_a$ . This simple heat balance yields the production-well temperature

$$T_p = \alpha T_i + (1 - \alpha) T_a \quad (5)$$

In response to thermal feedback, where the injection temperature  $T_i^{\text{tf}}$  remains constant, the substitution of Eq. (1) into Eq. (5) yields the steady-state production temperature  $T_p^{\text{tf}}$

$$T_p^{\text{tf}} = T_a + \alpha \Delta T(t=0) \quad , \quad T_i^{\text{tf}} = T_a + \Delta T(t=0) \quad (6)$$

Similarly, in response to thermal recycling, the steady-state production and injection temperatures are obtained by combining Eqs. (2) and (5), yielding

$$T_p^{\text{tr}} = T_a + \frac{\alpha}{1-\alpha} \Delta T \quad , \quad T_i^{\text{tr}} = T_a + \frac{1}{1-\alpha} \Delta T \quad (7)$$

With the knowledge of  $\alpha$  in Eq. (4), Eqs. (6) and (7) yield the purely advective best-case and worst-case temperature impact at the production well, respectively, and this allows quick and easy assessment of the system sustainability by comparison with the temperature range  $T_a \pm \Delta T_a$  for which the system was designed.

As an example, for a well doublet with an extraction/injection rate  $Q=70 \text{ m}^3/\text{day}$ , an aquifer thickness  $b=10 \text{ m}$ , a well separation distance  $L=20 \text{ m}$  and Darcy flux  $q=0.0432 \text{ m/day}$ , the return-flow criterion  $X=5.158 > 1$  is fulfilled, indicating that a recirculation cell develops with the return-flow fraction  $\alpha=0.46$ . Assuming a temperature difference  $\Delta T=-3 \text{ }^\circ\text{C}$  and the ambient temperature  $T_a=10 \text{ }^\circ\text{C}$ , the calculation yields the steady-state production temperatures  $T_p^{\text{tf}}=8.6 \text{ }^\circ\text{C}$  and  $T_p^{\text{tr}}=7.5 \text{ }^\circ\text{C}$  for thermal feedback and thermal recycling, respectively. Hence, if the system was designed for the operation range  $10 \text{ }^\circ\text{C} \pm 2 \text{ }^\circ\text{C}$ , one can see that the production temperature in response to thermal feedback is within this range, while the response to thermal recycling is not.

## From return-flow fraction ( $\alpha$ ) to capture probability ( $P_{ip}$ ) in arbitrary advective-dispersive systems

Thermal recycling in GWHP well doublets, as described in the previous section, is dictated by the heat flux evolution at an extraction well, which in turn is a function of the injected heat flux. In a well doublet, the relationship between the injected and the extracted heat flux can be defined by a transfer function (e.g. Jury and Roth 1990). The transfer function reflects the internal dynamics of an advective-dispersive system and is defined as the outlet response of the system to a Dirac input at inlet. A transfer function can also



be seen as a probability density function, reflecting the distribution of travel times between the two points.

The concept of transfer functions in hydrogeology is mostly used to address contaminant issues such as delimitation of well-head protection zones in complex three-dimensional (3D) systems (e.g. Frind et al. 2002) or to identify pollution sources (Neupauer and Wilson 2002; Milnes and Perrochet 2007). However, the analogy between the solute transport equation and the heat transport equation allows direct application of the same concepts to heat transport problems. The only difference which arises in heat transport processes is that interaction of a heat pulse with the rock matrix leads to retardation in the transit time distribution at the outlet. Scaling with the ratio between rock matrix and groundwater heat capacities (Gringarten and Sauty 1975; Lippmann and Tsang 1980), retardation is due to the initial heating of the surrounding rock matrix by the passing heat signal, followed by rock-mass cooling when the signal has passed. Therefore, retardation does not influence the capture probability of a heat parcel at the outlet.

The main assumption required for the transfer function theory to be applicable, is the principle of linear superposition, which states that the response of a system to a string of impulses is just the sum of the responses to the individual impulses. This assumption applies to heat transport under steady-state hydraulic conditions, in which injection of a fraction of a heat pulse at the inlet leads to simple scaling of the transfer function by that fraction at the outlet. With reference to thermal recycling, where the inlet and outlet fractions depend on each other, it will be seen in the following how the input signal can be directly expressed as a function of the output signal via a simple heat balance consideration.

Given a transfer function  $g(t)$ , the probability  $P_{ip}$  that whatever is introduced into the system at the injection well is captured at the production well is obtained by

$$P_{ip} = \int_0^{\infty} g(t) dt \quad (8)$$

Figure 2a illustrates the breakthrough curve obtained by a transient finite element simulation (FEFLOW) of heat transport between an injection and an extraction well, for a 100 m×100 m 2D horizontal aquifer with parameters shown in Fig. 2c. The longitudinal and transversal dispersivities for the simulations were chosen as  $\alpha_L=5$  m and  $\alpha_T=0.5$  m, respectively. The breakthrough curve reflects the transient evolution of the capture probability and is obtained by imposing a boundary condition  $P=1$  at the injection well and  $P=0$  along the inflowing limits, as described in Cornaton and Perrochet (2006a, b). The steady-state value corresponds to  $P_{ip}=0.32$ , reflecting the capture probability of the extraction well with respect to the injection well, according to Eq. (8). Differentiation of the breakthrough curve in Fig. 2a, yields the transfer function  $g(t)$ , as shown in Fig. 2b. Figure 2c illustrates the steady-state location probability field of the injection well, which can also be obtained by a single steady-state heat transport simulation.

The capture probability is equivalent to the fraction of the injected heat flux arriving at the production well, and is therefore the advective-dispersive counterpart of the return-flow fraction  $\alpha$  in Eq. (4). Using the hydraulic parameters given in Fig. 2c, the return-flow fraction  $\alpha=0.37$  is obtained, while the simulated capture probability is  $P_{ip}=0.32$ . This difference is due to the impact of dispersion/conduction and to the fact that the flow domain is of finite size, as opposed to the assumption made for the analytical solution in Eq. (4).

Transfer functions are fundamental to express output flux signals as functions of any given input flux  $I(t)$ . For a GWHP doublet, the output heat flux  $F_p(t)$  induced by return flow at the production well results from the convolution of the transfer function  $g(t)$  with the input heat flux  $I(t)$  at the injection well, namely

$$F_p(t) = Q_p \rho c (T_p(t) - T_a) = \int_0^t g(u) I(t-u) du \quad (9)$$

where  $\rho c$  is the specific heat capacity of water and  $Q_p$  the extraction rate.

Considering thermal feedback, according to Eq. (1), and knowing that the extraction rate  $Q_p$  is identical to the injection rate  $Q_i$ , the constant input heat flux is

$$I = Q_i \rho c (T_i - T_a) = Q_p \rho c \Delta T(t=0) \quad (10)$$

Introducing Eq. (10) into Eq. (9) leads to the production-well temperature in response to thermal feedback

$$T_p^{tf}(t) = T_a + \Delta T(t=0) \int_0^t g(u) du \quad (11)$$

Equation (11) reveals that the convolution integral in Eq. (9) reduces to a simple integration of the transfer function  $g(t)$ , and, in conjunction with Eq. (8), that the steady-state temperature at the production well is simply

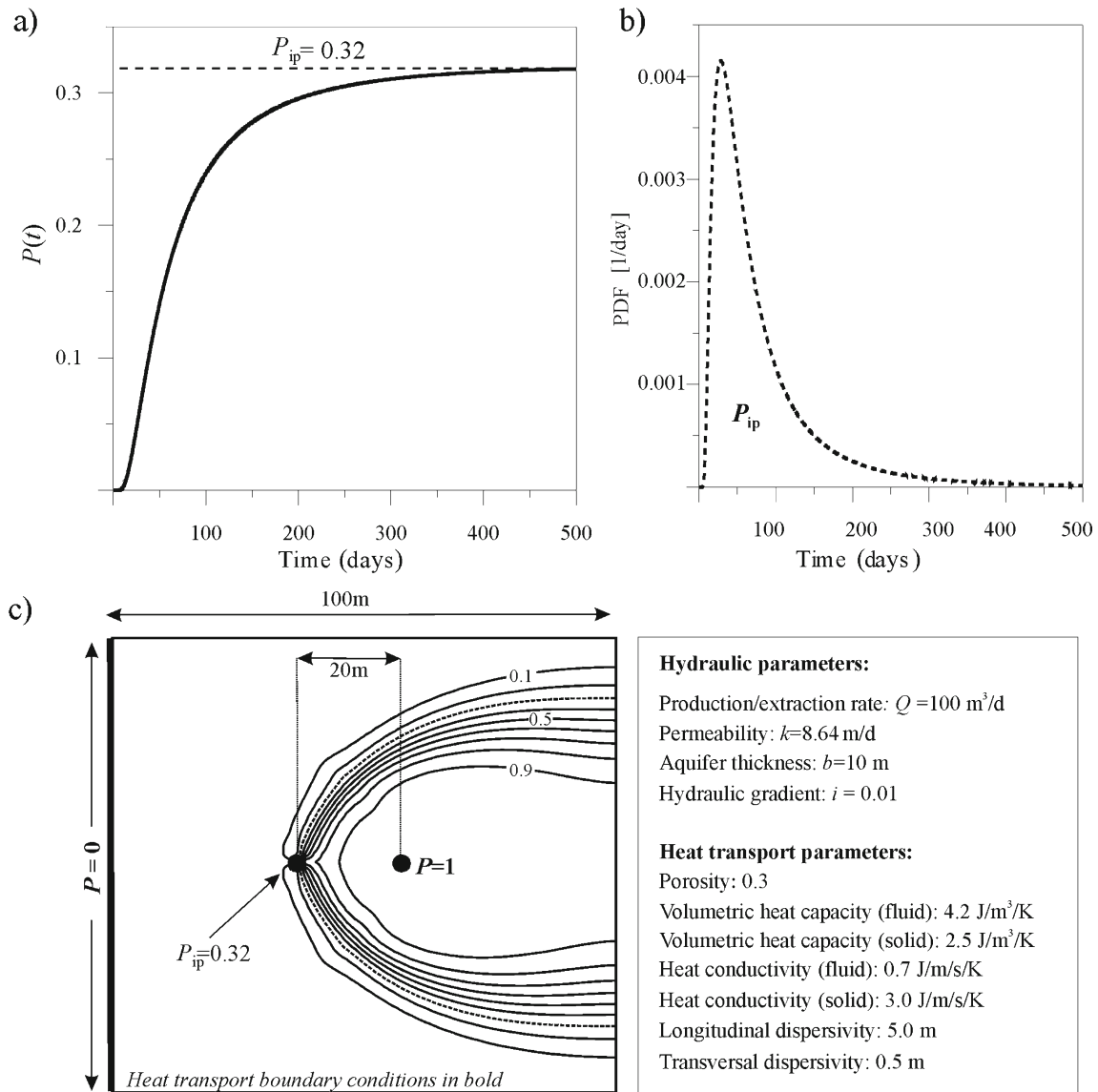
$$T_p^{tf} = T_a + P_{ip} \Delta T(t=0) \quad (12)$$

Considering thermal recycling with a constant temperature difference between the injection and production wells, as indicated in Eq. (2), leads to the transient input heat flux

$$I(t) = Q_i \rho c (T_i(t) - T_a) = Q_p \rho c (T_p(t) + \Delta T - T_a) \quad (13)$$

Introducing Eq. (13) into Eq. (9) yields the evolution of the temperature at the production well

$$T_p^{tr}(t) = T_a + \int_0^t g(u) T_p^{tr}(t-u) du + (\Delta T - T_a) \int_0^t g(u) du \quad (14)$$



**Fig. 2** Numerical simulations carried out on a 2D horizontal aquifer with the same flow field as shown in Fig. 1, and with parameters indicated (c). **a** Breakthrough curve obtained by transient simulation of heat transport with a boundary condition of  $P=1$  at the injection well, yielding the capture probability  $P_{ip}$  at steady state, reflecting Eq. (9). **b** Transfer function  $g(t)$ , also called probability density function (PDF), obtained by derivation of the breakthrough curve  $P(t)$  (a). **c** Steady-state location probability field of the injection well; heat boundary conditions indicated in **bold**

Transforming Eq. (14) into Laplace space, in which convolution integrals reduce to simple products, results in

$$\begin{aligned}\widehat{T}_p^{\text{tr}}(s) &= \frac{T_a}{s} + \widehat{g}(s)\widehat{T}_p^{\text{tr}}(s) + (\Delta T - T_a)\frac{\widehat{g}(s)}{s} \\ &= \frac{T_a}{s} + \frac{\widehat{g}(s)}{s(1 - \widehat{g}(s))}\Delta T\end{aligned}\quad (15)$$

where  $s$  stands for the Laplace variable and the circumflex indicates the transformed functions.

Given that (see for details Milnes and Perrochet 2006)

$$T_p^{\text{tr}}(t = \infty) = \lim_{s \rightarrow 0} s\widehat{T}_p^{\text{tr}}(s) \quad , \quad \lim_{s \rightarrow 0} \widehat{g}(s) = \int_0^\infty g(t) dt = P_{ip} \quad (16)$$

the steady-state production temperature accounting for thermal recycling is

$$T_p^{\text{tr}} = T_a + \frac{P_{ip}}{1 - P_{ip}}\Delta T \quad (17)$$

while, according to Eq. (2), the steady-state injection temperature is

$$T_i^{\text{tr}} = T_a + \frac{1}{1 - P_{\text{ip}}} \Delta T \quad (18)$$

It is noted here that the production temperatures in Eqs. (12) and (17) which include the effects of thermal advection, dispersion and conduction, are in perfect analogy with Eqs. (6) and (7) which were derived from a purely advective heat-flux balance at the production well. The notion of return flow evaluated by the purely hydrodynamic fraction  $\alpha$  is thereby generalized to the return-heat-flow fraction, or capture probability  $P_{\text{ip}}$ , dictated by the full heat transport process.

For the model shown in Fig. 2, with the capture probability  $P_{\text{ip}}=0.32$  and the corresponding return flow fraction  $\alpha=0.37$ , an ambient groundwater temperature  $T_a=10^\circ\text{C}$  along with a temperature difference  $\Delta T=+3^\circ\text{C}$  across the heat exchanger lead to the following steady-state production and injection temperatures for the two processes.

#### *Thermal feedback*

Advective-dispersive,

Eqs. (12) and (1) :  $T_p^{\text{tf}} = 10.96^\circ\text{C}$  ,  $T_i^{\text{tf}} = 13^\circ\text{C}$

Hydrodynamic,

Eq. (6) :  $T_p^{\text{tf}} = 11.11^\circ\text{C}$  ,  $T_i^{\text{tf}} = 13^\circ\text{C}$

#### *Thermal recycling*

Advective-dispersive,

Eqs. (17) and (18) :  $T_p^{\text{tr}} = 11.41^\circ\text{C}$  ,  $T_i^{\text{tr}} = 14.41^\circ\text{C}$

Hydrodynamic,

Eq. (7) :  $T_p^{\text{tr}} = 11.76^\circ\text{C}$  ,  $T_i^{\text{tr}} = 14.76^\circ\text{C}$

Injection-well temperatures, as calculated in the preceding for advective-dispersive thermal feedback and recycling, can be directly used as temperature boundary conditions in steady-state heat-transport models. While  $T_i^{\text{tf}}$  is commonly used in classical modelling of GWHP systems (e.g. Lo Russo and Civita 2009), leading to the best-case thermal impact on the aquifer (or worst-case heat load requirement),  $T_i^{\text{tr}}$  now also allows direct simulation of the corresponding worst-case thermal impact (or best-case heat-load requirement). For the model example given in Fig. 2, Fig. 3 illustrates the respective steady-state advective-dispersive thermal plumes resulting from the above temperature boundary conditions at the injection wells for the two processes.

Figure 3a shows the thermal plume arising from thermal feedback, reflecting the classical approach (best-case thermal impact), and Fig. 3b reveals the plume arising from thermal recycling (worst-case thermal impact). The

heat plume arising from thermal recycling is significantly more developed than for thermal feedback. Together, these two simulations define the end-members to be considered also when assessing the downstream potential impacts. As an example, Swiss legislation requires that the thermal impact on the groundwater must not exceed  $\pm 3^\circ\text{C}$  of the ambient temperature  $T_a$  at a distance further than 100 m downstream of a well doublet (OFEV 2009). In the example in this paper, the plume arising from the best-case thermal impact shown in Fig. 3a for thermal feedback indicates that the temperature impact does not exceed the legal limit. However, thermal recycling induces a temperature increase at the injection well which leads to a plume with temperatures exceeding the limit far downstream.

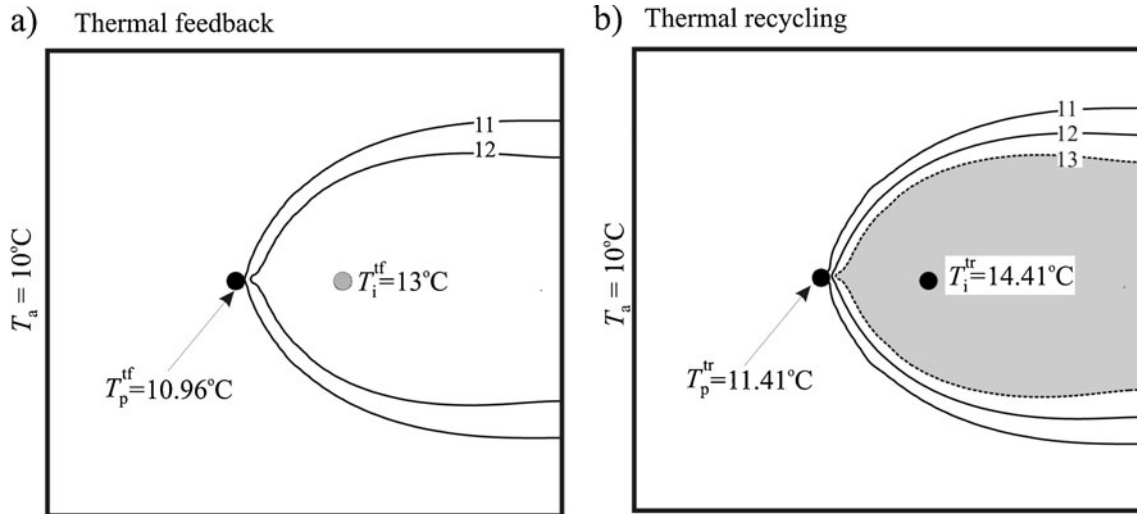
## Discussion and conclusions

The main contribution of this work lies in the mathematical description of thermal recycling, as opposed to the well-known process of thermal feedback. The approach is based on the assumption of a constant  $\Delta T$  across the heat exchanger. Even though this assumption is not fulfilled during operation of GWHP systems, it allows identification of the worst-case as opposed to the best-case thermal impact given by thermal feedback, commonly used during the design phase of GWHP systems. The conjunctive consideration of these two end-members therefore yields a novel design approach, allowing assessment of the sustainability of a system with respect to its efficiency and to a range of potential impacts.

The analytical solution presented allows determination of the hydrodynamic return flow fraction  $\alpha$  in a well doublet. It was shown that steady-state production and injection temperatures in response to both thermal feedback and thermal recycling, can be expressed by this parameter, allowing quick assessment of the asymptotic behaviour of the system for multiple parameter sets and hydraulic conditions.

Based on the transfer function theory, the approach was extended to advective-dispersive systems. It revealed that the steady-state production temperatures for thermal feedback and recycling are functions of the capture probability  $P_{\text{ip}}$ , which can be obtained by a single steady-state heat-transport simulation for arbitrary well-doublet configurations. Simulation of the steady-state plumes allows assessment of the impact of a GWHP system on the aquifer for the two end-member processes (thermal feedback and thermal recycling).

Since a steady-state thermal plume will change the ambient temperature in the downstream area, the approach can easily be extended to assess the sustainability of successive downstream well doublets. To do so, the production-well temperature calculations have to be carried out successively, from the upstream towards the



**Fig. 3** Steady-state thermal plumes obtained for the 2D horizontal aquifer shown in Fig. 2: iso-values given in °C; both simulations were run with the inflowing western heat-boundary condition of  $T_a = 10^\circ\text{C}$ ; **a** Steady-state thermal plume in response to thermal feedback with an injection-well heat boundary condition of  $13^\circ\text{C}$ , according to Eq. (1); **c** Steady-state thermal plume in response to thermal recycling with an injection-well temperature boundary condition of  $14.41^\circ\text{C}$ , according to Eq. (18)

downstream located well doublets, by successive adjustment of the ambient groundwater temperature of each well doublet, reflecting the net temperature effect induced by the upstream well doublets.

The main limitation of the presented approach is related to the steady-state conditions, which in no way reflect such highly transient systems. It is well known that GWHP installations are exploited on a seasonal basis, with even daily and weekly variations. On the one hand, the restrictive assumption of steady-state conditions is alleviated by the fact that the approach allows quick assessment of the asymptotic behaviour of a system in response to multiple parameters sets and various hydrodynamic conditions. This quick assessment approach may then allow identification of potentially viable conditions which can then be used for in-depth transient analysis. On the other hand, a steady-state approach will yield the most conservative results, which is often a criterion for the design of such systems.

Many GWHP design tools are based on such steady-state considerations, raising the crucial question of how steady-state conditions should be reasonably defined for such dynamic systems. A major challenge for future work in this field is therefore related to the identification of dynamic steady-state conditions arising from heating/cooling cycles with variable hydraulic conditions and how these translate to average steady-state conditions. If a way is found to describe dynamic steady-state conditions in terms of an average steady-state regime reflecting the long-term transient behaviour of seasonal exploitation, then the efficiency of the presented approach would be considerably increased.

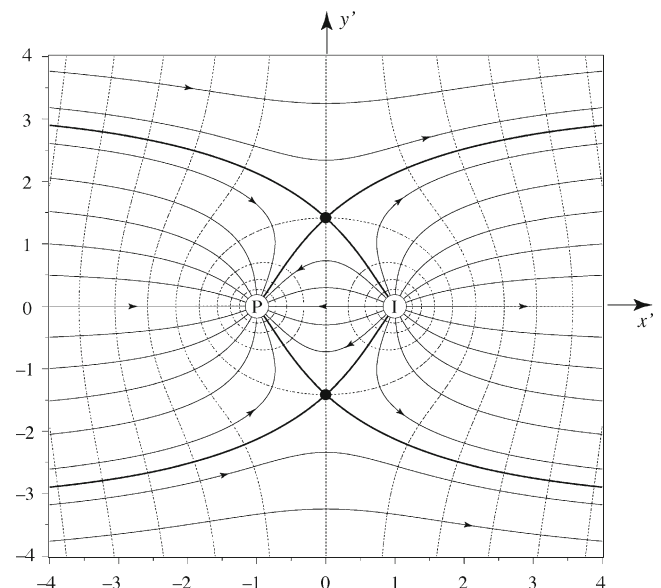
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### Appendix: Characteristic parameters of a well doublet with partial recirculation in an aquifer with uniform regional flow

The complex potential,  $\Omega$ , resulting from the implementation of a well doublet parallel to a 2D uniform flow field in the  $(x, y)$ -plane, with the injecting well located downstream, is classically obtained (e.g. Strack 1989; Javandel and Tsang 1986; Luo and Kitanidis 2004) by the superposition

$$\Omega = \frac{Q}{2\pi b} \ln(z + d) - \frac{Q}{2\pi b} \ln(z - d) - qz, \quad (19)$$

$$z = x + iy$$



**Fig. 4** Dimensionless complex plane representation with pumping (P) and injecting (I) wells located at  $z' = \pm 1$ , equipotentials (dashed), streamlines (solid), stagnation points (dots) and flow separation lines (bold). Partial recirculation patterns for  $X=3$



where  $b$  is the aquifer thickness and  $z$  is the coordinate in the complex plane. The first two terms correspond, respectively, to the flow rate  $Q$  extracted at  $z=-d$  and to the same flow rate injected ( $-Q$ ) at  $z=d$ , while the third term accounts for the regional uniform Darcy flux  $q$  defined positive in the direction of the real axis (Fig. 4). The two wells are therefore centred about the origin along the  $x$ -axis and are separated by the distance  $L=2d$ .

The complex function above features the hydraulic potential  $h$  and the stream function  $\psi$  as real and imaginary parts, respectively, so that

$$\Omega = k h + i \psi \quad (20)$$

where  $k$  is the hydraulic conductivity of the medium.

Using the dimensionless variables

$$\begin{aligned} \Omega' &= \frac{b\Omega}{Q}, \quad z' = \frac{z}{d}, \quad X = \frac{Q}{\pi b q d}, \\ h' &= \frac{b k h}{Q}, \quad \psi' = \frac{b \psi}{Q} \end{aligned} \quad (21)$$

the complex potential becomes

$$\begin{aligned} \Omega' &= h' + i\psi' = \frac{1}{2\pi} \ln\left(\frac{z'+1}{z'-1}\right) - \frac{z'}{\pi X}, \\ z' &= x' + i y' \end{aligned} \quad (22)$$

and is illustrated in Fig. 4, where the dimensionless hydraulic potential  $h'$  and stream function  $\psi'$  are represented for the dimensionless flow rate  $X=3$ .

According to the complex potential theory, the dimensionless specific discharge is obtained by the differentiation

$$W' = -\frac{d\Omega'}{dz'} = \frac{1}{\pi} \left( \frac{1}{z'^2 - 1} + \frac{1}{X} \right) \quad (23)$$

This function has no complex components at either purely real or purely imaginary locations. It is singular at the well points and vanishes at the stagnation points  $z'_s$ .

Setting  $W''(z'_s)=0$  in (Eq. 23) yields

$$z'_s = \pm \sqrt{1-X}, \quad X < 1 \quad (24)$$

$$z'_s = \pm i\sqrt{X-1}, \quad X > 1 \quad (25)$$

Hence, as long as  $X < 1$ , two stagnation points are located on the real axis ( $y'=0$ ) and there is no recirculation from the injection well to the pumping well. As  $X$  increases, the stagnation points shift towards the origin, merge at  $z'_s = 0$  when  $X=1$ , and then shift away along the imaginary axis ( $x'=0$ ), as indicated by (Eq. 25) for  $X > 1$ . In true dimensions, the stagnation points are located at  $x=0$  and  $y = \pm d\sqrt{X-1}$ .

In this latter case, a recirculation cell develops and a fraction of the injected flow rate returns to the pumping well.

The shortest travel time of the recycled water particles is the hydraulic breakthrough-time, dictated by the inverse of the specific discharge along the straight flow-path connecting the two wells. Given that specific discharge has no complex components along this path, this characteristic time is easily obtained, in dimensionless form, by

$$t'_{min} = \int_1^{-1} \frac{1}{W'} dz' = 2\pi X \left( \frac{X}{\sqrt{X-1}} \tan^{-1}\left(\frac{1}{\sqrt{X-1}}\right) - 1 \right) \quad (26)$$

with the limit value  $t'_{min} = 4\pi/3$  when  $X \rightarrow \infty$ .

In true dimensions,  $t_{min} = t'_{min} \phi b d^2 / Q$  where  $\phi$  is the porosity of the medium.

The fraction of the flow rate recycled in the well doublet is expected to increase with the flow rate itself. Under increasing recycling conditions, the stagnation points (Eq. 25) are always at purely imaginary locations and connected by a straight (equipotential) line intersecting all recirculating flow-paths. As specific discharge always remains real-valued along this line (Eq. 23), the fraction recycled is therefore straightforwardly obtained by

$$\alpha = i \int_{-i\sqrt{X-1}}^{i\sqrt{X-1}} W' dz' = \frac{2}{\pi} \left( \tan^{-1}(\sqrt{X-1}) - \frac{\sqrt{X-1}}{X} \right) \quad (27)$$

## References

- Banks D (2007) Thermogeological assessment of open loop well doublet schemes: an analytical approach. Proc. 27th Annual Conference of the Irish Group of the International Association of Hydrogeologists, Tullamore, Ireland, 24–25 April 2007
- Banks D (2008) An introduction to thermogeology: ground source heating and cooling. Blackwell, Oxford, 339 pp
- Banks D (2009) Thermogeological assessment of open-loop well-doublet schemes: a review and synthesis of analytical approaches. *Hydrogeol J* 17:1149–1155
- Clyde CG, Madabhushi GV (1983) Spacing of wells for heat pumps. *J Water Resour Plan Manage* 109(3):203–212
- Cornaton F, Perrochet P (2006a) Groundwater age, life expectancy and transit time distributions in advective-dispersive systems: 1. generalized reservoir theory. *Adv Water Res* 29(9):1267–1291
- Cornaton F, Perrochet P (2006b) Groundwater age, life expectancy and transit time distributions in advective-dispersive systems: 2. reservoir theory for sub-drainage basins. *Adv Water Res* 29(9):1292–1305
- Ferguson G (2006) Potential use of particle tracking in the analysis of low-temperature geothermal developments. *Geothermics* 35:44–58
- Frind EO, Muhammad DS, Molson JW (2002) Delineation of three-dimensional well capture zones for complex multi-aquifer systems. *Ground Water* 40(6):586–598
- Gringarten AC, Sauty JP (1975) A theoretical study of heat extraction from aquifers with uniform regional flow. *J Geophys Res* 80:4956–4962
- Javandel I, Tsang CF (1986) Capture-zone type curves: a tool for aquifer cleanup. *Ground Water* 24(5):616–625
- Jury WA, Roth K (1990) Transfer functions and solute movement through soil: theory and applications, Birkhäuser, 226 pp

- Lippmann MJ, Tsang CF (1980) Groundwater use for cooling: associated aquifer temperature changes. *Ground Water* 18 (5):452–458
- Lo Russo S, Civita MV (2009) Open-loop groundwater heat pumps development for large buildings: a case study. *Geothermics* 38:335–345
- Luo J, Kitanidis PK (2004) Fluid residence times within a recirculation zone created by an extraction-injection well pair. *J Hydrol* 295:149–162
- Milnes E, Perrochet P (2006) Direct simulation of solute recycling in irrigated areas. *Adv Water Res* 29(8):1140–1154
- Milnes E, Perrochet P (2007) Simultaneous identification of a single pollution point-source location and contamination time under known flow field conditions. *Adv Water Res* 30(12):2439–2446
- Neupauer RM, Wilson JL (2002) Backward probabilistic model of groundwater contamination in non-uniform and transient flow. *Water Resour Res* 25(7):733–746
- OFEV (2009) Exploitation de la chaleur tirée du du sous-sol. Aide à l'exécution destinée aux autorités et aux spécialistes de géothermie [Exploitation of heat extracted from the ground. Guidelines for administrations and specialists in the field of geothermics]. Published by Office Fédéral de l'Environnement, Bern, Switzerland. Available via <http://www.bafu.admin.ch/publikationen/publikation/01042/index.html?lang=fr>. Accessed 20 July 2012
- Rafferty KD (1996) Well-pumping issues in commercial groundwater heat pump systems. *ASHRAE Trans* 104(1B):927–931
- Strack O (1989) *Groundwater Mechanics*, Prentice-Hall, Upper Saddle River, NJ, 732 pp
- Warner DL, Algan U (1984) Thermal impact of residential groundwater heat pumps. *Ground Water* 22(1):6–12
- Zhao Y, Zhang SG, Xun L (2003) Cost-effective optimal design of groundwater source heat pumps. *Appl Geotherm Eng* 23 (13):1595–1603